WORLD DISTRIBUTION OF MEAN MONTHLY AND ANNUAL PRECIPITABLE WATER 1

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ABSTRACT

Mean values of precipitable water are determined for 182 stations throughout the world using a 3-yr. period and evaluation methods developed by Peterson and Showalter. The resulting distribution is presented on a series of monthly maps and a map of the annual mean value. A general latitudinal pattern is present, especially over the oceans, with Highs near the Equator and Lows at the Poles. The continents show the effects of elevation and distance from the sea. The highest yearly mean is found in the Bay of Bengal in the summer. Other areas of high precipitable water are the tropical lowlands. Lowest values are in the high latitudes, continental interiors, and at high elevations in winter. Annual ranges are greatest in the Bay of Bengal and interior Asia and lowest over the equatorial oceans.

1. INTRODUCTION

Precipitable water is defined as the depth of liquid water that would be obtained if the total amount of water vapor in a specified layer of the atmosphere above a unit area of the earth's surface were condensed into a layer on that surface. This concept has found major applications in quantitative precipitation forecasting, determination of the moisture flow over an area, and especially in radiation balance studies. In spite of its uses and its familiarity to meteorologists, only limited attempts have been made to determine and map the distribution of mean precipitable water vapor. Many of the published data on precipitable water values are found only as a minor element in works dealing with other problems, such as quantitative forecasting or moisture flow [8, 9, 11, 22]. Studies dealing directly with precipitable water usually concentrate on a limited area [1, 2, 13, 15, 17]. Mapping of the distribution has been attempted only for limited areas or on an annual or seasonal basis. Shands [19] and Reitan [16] present a series of monthly maps and an annual map covering the United States. Crisi [5] mapped the annual distribution in the Northern Hemisphere for the year 1958. Bannon and Steele [3] covered the world between the latitudes 70°N. and 52°S., presenting maps for the midseason months of January, April, July, and October.

The purpose of the current study is to fill some of the gaps in the present literature on precipitable water vapor. The distribution of precipitable water is mapped using a worldwide coverage, including both hemispheres and extending farther poleward than previous studies. Maps are constructed for each month and for the annual mean value. More current data and an expanded station

coverage are now available. Finally, the presentation of data from this study will do much to confirm or deny the accuracy of former studies.

2. METHODOLOGY

Two methods of evaluation were employed to determine the values of precipitable water. Both are simple to use, require only data that are readily available in several widely distributed publications, and provide an accuracy that is reasonable within the scale of the current study. The two methods are a precipitable water nomogram devised by Peterson [14] and a transparent template developed by Showalter [20].

In using these methods the required data are the surface pressure in millibars and the dew point temperatures in degrees Celsius for the surface, 850 mb., 700 mb., and 500 mb. For the current study these were obtained from ESSA's Monthly Climatic Data for the World on a mean monthly basis. The use of mean monthly dew points leads to an underestimate of mean specific humidity and, therefore, precipitable water. This is because the relationship between dew point and specific humidity is not linear. The differences are small, however, and within the scale of the present study are not significant. Bannon and Steele [3] found that the use of mean monthly dew points to calculate the mixing ratio underestimated the true value by about 4 percent in most areas and up to 8 percent in areas with great variability in humidity (based on an analysis of data for Lerwick, Scotland; Gibraltar; and Aden). In precipitable water evaluation, this underestimate is offset by overestimates due to the lag errors of upper air humidity instruments (1 percent) and the use of data at widely spaced pressure levels (850 mb., 700 mb., and 500 mb.) to approximate specific humidity within these

¹ This paper is the condensation of a Master's thesis in geography submitted at the University of California, Los Angeles, December 1967, under the direction of Dr. Werner Terjung, Dr. Jonathan Sauer, and Dr. Morris Neiburger.

Table 1.—Comparison of precipitable water computed from mean monthly dew point temperatures with the monthly mean of daily soundings

Station	Month	Range of dew point tempera- tures at 850 mb.	Precipitable Water Vapor	
			From mean monthly dew points	Mean of daily soundings
Oklahoma City	Jan.	33° C.	0. 22 in.	0.25 in.
Oklahoma City	Apr.	32°	. 60	. 65
Oklahoma City	July	13°	1, 38	1.41
Oklahoma City	Oct.	20°	. 81	. 83
Sault Ste. Marie	Apr.	35°	. 38	. 40
Sault Ste. Marie	Oct.	34°	. 68	. 69
Miami	Jan.	26°	. 96	1.02
Miami	July	11°	1.66	1.69
Point Barrow	July	21°	. 58	. 60

layers (2 percent to 5 percent). Bannon and Steele estimate a net underestimate of 1 to 3 percent in precipitable water evaluated by means similar to those used in this study. A comparison of precipitable water values computed from mean monthly dew points and from individual daily soundings is presented in table 1. The results here also indicate that the error involved in using mean monthly data is small and for a study of this scale should cause no concern.

Peterson's nomogram was used in over 90 percent of the calculations. It was preferred over the Showalter template because it is more rapid, easier to use, and calls for less subjective estimation on the part of the user. The basic assumption used in the construction of the nomogram is that the dew point varies linearly between the successive pressure levels employed. This is considered to be a valid assumption for higher values of precipitable water but not necessarily for lower amounts, especially below 0.50 in. Peterson gives no estimate of the accuracy of his nomogram. As part of the present project, a sample study, using a scattering of stations in the United States for which precipitable water data are available from a study by Reitan [17], was carried out to check the reliability of the nomogram. Monthly values for stations in differing environments and for a variety of seasons were selected. Reitan's results were compared with precipitable water values computed from the nomogram. The values agreed to within about 5 percent for the whole range of stations. Greatest percentage variations are found where values of precipitable water are low. Here the nomogram results are a little over 10 percent from Reitan's. Figure 1 shows a scattergram comparing the results of the two methods.

The Peterson nomogram was not applicable to all the stations or months used in this study. It is designed only for stations where the surface pressure is greater than 940 mb. and the surface dew point temperature is above —5°C. In cases where these conditions were not met the Showalter template was used. To obtain the amount of precipitable water, dew point temperatures are plotted

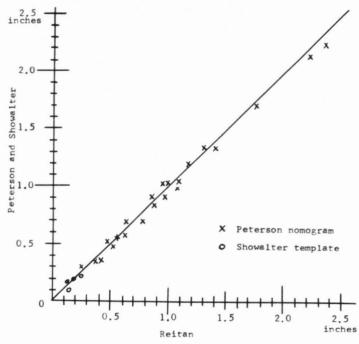


FIGURE 1.—The comparison of the values of precipitable water calculated with the Peterson nomogram and Showalter template with those published by Reitan for the same month and year.

on a pseudoadiabatic chart. The amount of precipitable water is then read by means of slanting lines on the transparent template for each pressure layer. The amounts are totaled to give the final result. Accuracy, as estimated by Showalter, is within 5 percent for high values of precipitable water. This decreases to an average error of about 10 percent when precipitable water is low. Since the template was used for high altitude and low temperature stations, where the values of precipitable water are low, accuracy in the current study is closer to the latter figure. Results for four stations have been included on the scattergram in figure 1 to give some idea of how the template compares with Reitan in low precipitable water situations.

Since the present study considers the layer from the surface to 500 mb. it somewhat underestimates the total water vapor content above an area. In practical applications, however, the amount of precipitable water above this level is usually considered inconsequential.²

The period selected for the study was 1964–1966. While a longer period might have yielded more precise averages, the 3-yr. period was selected because the upper air data published in *Monthly Climatic Data for the World* becomes increasingly sketchy with each preceding year. Both the number of stations reporting and the data presented for each becomes less complete.

In addition to the 1964-1966 data, records from 1967 were available for January, February, March, and April.

² Most studies employ either the 500-mb., 5-km., or 400-mb. level as the upper limit. Bannon and Steele [3] show the amount of precipitable water above the 500-mb., level to range from about 0.0008 in. near the Poles in winter to a maximum of 0.24 in. in the Bay of Bengal in summer. It is less than 10 percent of the total in the surface to 500-mb. level, but becomes more critical for high altitude stations.

STATIONS

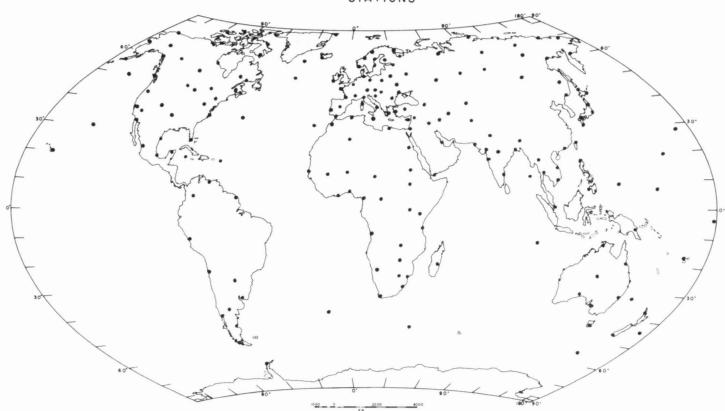


FIGURE 2.—Stations selected for the current study.

Values for these months were used to give 4-yr. averages for stations where the 1964–1966 values showed a fairly wide range. Where available, 1963 data were used for the same purpose for May through December. Where data from 1 of the yr. in the 1964–1966 period were thought questionable, the supplementary data were used as a check. Several new stations were added to the published data in the 1967 issues, especially in the lightly covered areas such as South America. Values were calculated and used to check the interpolated isolines in these areas.

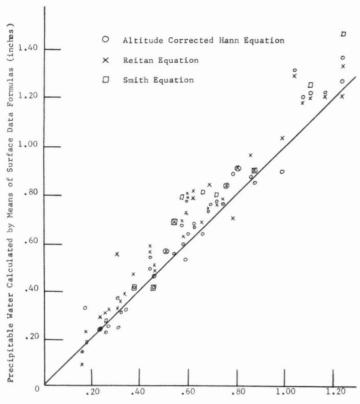
One hundred eighty-two full record and 18 supplementary stations were selected for the current study. Full record stations range in latitude from 81°36′N. to 65°15′S. Of the supplementary stations, eight had from 1 to 3 yr. of record and 10 had less than 1 yr. These stations were used as checks on isoline placement and represent stations in otherwise lightly covered areas that first appeared in the upper air section of *Monthly Climatic Data for the World* since 1964.

Oceanic areas are covered by island stations, weather ships and sea level stations on the margins of continents. In choosing these stations an attempt was made to select those that more clearly represented the sea surface itself than a particular topographic situation unique to some land location. Land stations were selected to sample as many different combinations of elevation, climatic type, and continental or maritime location as possible. Areas

where precipitable water mapping had been done before received less attention than they might otherwise merit. Figure 2 shows the resulting station network.

Certain regions of the world are largely lacking in published upper air data. Central America and China are prime examples. South America, interior Africa, and much of southwest Asia have only sparse coverage. In such regions three techniques were used to estimate the values of precipitable water and to draw the isolines. Wherever possible they were surrounded by as dense a station network as practical. Estimates were made for several points utilizing values calculated for stations in roughly similar environments in other parts of the world. Finally, an approximate calculation of precipitable water was made using surface moisture data. The latter was used to check the magnitude of the interpolated isolines and was not considered as an accurate station value.

Estimation of precipitable water by equations employing a surface moisture parameter has long been used. Basically these equations assume a specific vertical moisture profile so cannot be considered truly accurate unless some special condition prevails, such as a pseudoadiabatic lapse rate. Nevertheless, many authors [4, 6, 10, 12, 18, 21] consider surface conditions to be an adequate indicator of total precipitable water. Equations of this type are often used, therefore, in regions where moisture data for the upper levels are not available.



Precipitable Water Calculated from Radiosonde Data (inches)

Figure 3.—A comparison of values of precipitable water computed by means of equations employing surface data with those calculated by the Peterson nomogram and Showalter template using upper air data.

Figure 3 shows the comparison of the values of precipitable water calculated by means of three equations employing surface data with the values calculated from radiosonde data. The first equation is a modification of Hann's [7] simple expression to take into account the decrease of precipitable water with elevation. It is frequently applied [6, 10, 12] and is the one utilized in this study. The equation uses surface vapor pressure and has the form:

$$W=2.3e10^{-t/22,000}$$

where W is precipitable water, e is surface vapor pressure, and z is the elevation of the station in meters.

A recent equation that relates the natural logarithm of precipitable water to the surface dew point has been developed by Reitan [18] from empirical data from 15 United States stations. It is in the form:

$$\ln W = -0.981 + 0.0341D$$

where D is the dew point in ${}^{\circ}F$, and W is in centimeters.

Smith [21] proposes a variable coefficient to make Reitan's equation more appropriate for all latitudes and seasons. He presents a table for deriving the coefficient for 10° lat. belts in the Northern Hemisphere.

The scattergram in figure 3 was constructed using 41 values of precipitable water from 13 stations throughout

the world. These range in altitude from sea level to 2547 m. and represent a variety of environmental conditions. All 41 values were used to compare the "altitude-corrected" Hann equation and the Reitan equation with values calculated from upper air data. Ten values from Northern Hemisphere stations were used to test the Smith equation. The graph shows that the surface equations generally tend to overestimate the precipitable water content of the atmosphere. The "altitude-corrected" Hann equation seems to give the best overall results, primarily due to the fact that it does take into account the elevation of the station, an important factor in determining the amount of precipitable water. For this reason, the "altitude-corrected" Hann equation was used in this study.

Average values of precipitable water vapor for each month at each station were computed and the average of the monthly means was taken to provide an annual mean. These were plotted on a Hammer's Equal Area projection. Isolines of precipitable water were drawn for each 0.25 in. At this scale of mapping only the broad general pattern can appear. Much of the local variation had to be eliminated. This is especially true of mountain areas. Smaller mountain chains cannot be shown on the scale employed and only the general trend toward low values is shown for the major mountain systems.

3. SEASONAL DISTRIBUTION

Consideration of the world distribution of precipitable water vapor can best begin by looking at the annual map (fig. 4). The general latitudinal pattern can be seen by observing the oceanic areas. Lowest values are found in the polar regions. The amount of moisture in the air increases steadily with decreasing latitude, reaching to over 1.75 in. in the equatorial regions. This oceanic latitudinal pattern shows the dependence of precipitable water on temperature. The zone of highest mean annual precipitable water over the oceans, however, is not located symmetrically around the geographic Equator, nor is it symmetric around the so-called "heat equator." The 1.75-in. zone extends from 5°N. to 11°S. with a northward extension in the Indian Ocean and the Bay of Bengal.

A perfectly latitudinal pattern is somewhat disrupted by two factors, however. The first is the ocean currents and their differing temperature regimes, while the second is the effect of the subsidence inversion at the eastern ends of the subtropical high pressure cells in lowering the precipitable water content in these regions.³ Because atmospheric moisture drops rapidly through the inversion layer, the assumption of a linear moisture depletion profile between widely separated pressure levels, used in the computation for this study, is not valid. The result would be an overestimation of precipitable water in these areas. The subsidence inversion is particularly strong over the eastern ends of the Hawaiian and Azores Highs in summer

³ Personal communication from Prof. Morris Neiburger. The author wishes to thank Prof. Neiburger for his aid and suggestions in dealing with these critical areas.

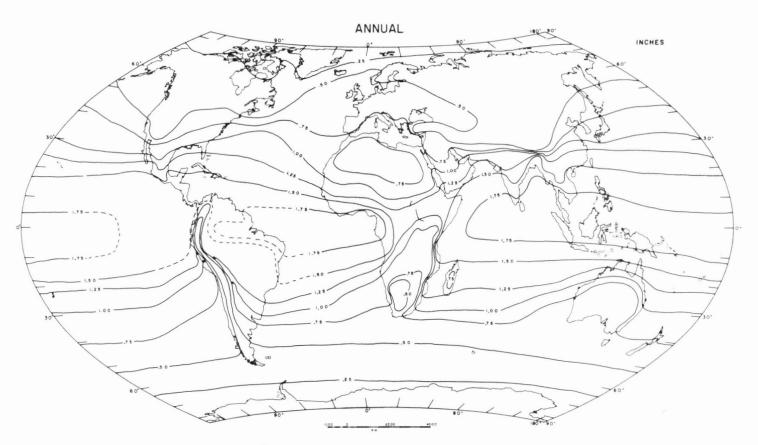


FIGURE 4.—Mean precipitable water—annual.

and to a lesser degree over the three high pressure cells in the Southern Hemisphere during the entire year. In these regions the effects of the inversion in lowering the values of precipitable water have been taken into account.

The effect of land and elevation in lowering the amount of precipitable water can be seen by the equatorward bending of the isolines over the continents. This is well illustrated in North America, the Andes of South America, interior Australia, interior Asia, and the African plateau. The lowest values of mean annual precipitable water for the world are found at the Poles. In the middle latitudes, the lowest means are found over mountain ranges and interior Asia. The highest amounts of precipitable water over land areas are found in the tropical lowlands such as the Amazon and Congo Basins. In general, areas of low precipitable water over the continents are associated with low temperatures, continental interiors far from oceanic sources of moisture, and/or at high elevations. Areas of high annual precipitable water can be observed at low elevations, with high mean annual temperatures, and close proximity to large sources of advected moisture.

Elevation accounts for the most radical departures from the latitudinal arrangement and for the greater local diversity over the continents. One effect of mountains, that of barriers, can be seen by comparing North America and Europe. The north-south oriented mountains in North America cause the isolines of precipitable water to swing sharply southward only a short distance from the west coast. In Europe, where the mountains are oriented east-west, a rather uniform value of precipitable water is seen to penetrate far inland. This causes a more uniform longitudinal distribution and higher amounts over the European area.

It has often been said that the air over deserts is not really dry and these areas lack precipitation primarily because of their high saturation deficits and stable atmospheric conditions. This is confirmed by the annual map of precipitable water. The Sahara, for example, has nearly as much precipitable water as the cooler regions of northern Europe and the northern United States. The deserts do have less precipitable water than other areas in the same latitudes, however.

Examination of both the annual and the monthly maps shows some basic differences between the Northern and Southern Hemispheres, largely due to their differing distributions of land and water. The Southern Hemisphere exhibits a more regular latitudinal pattern. In the Northern Hemisphere this is broken by the presence of the large landmasses. The effect is seen not only over the continents themselves, but also in the oceanic areas. Here, the more

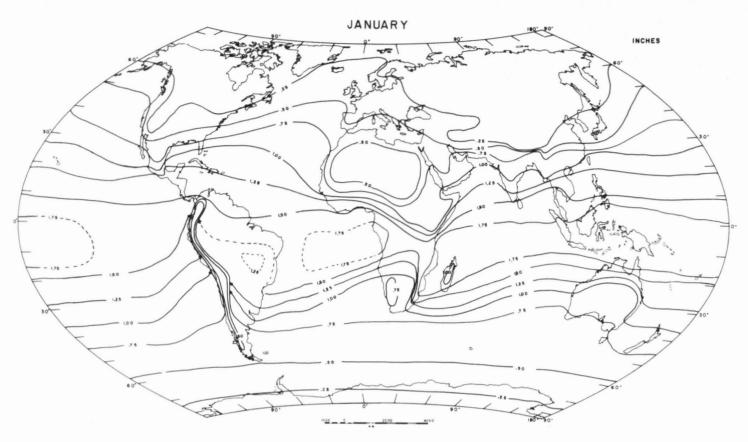


FIGURE 5.—Mean precipitable water—January.

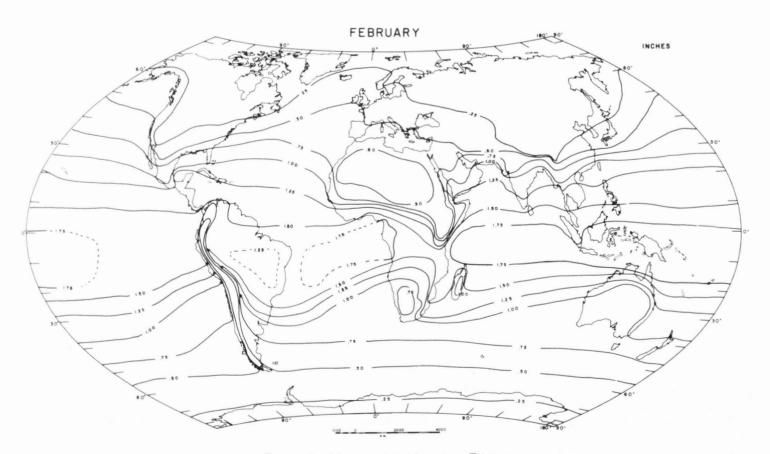


Figure 6.—Mean precipitable water—February.

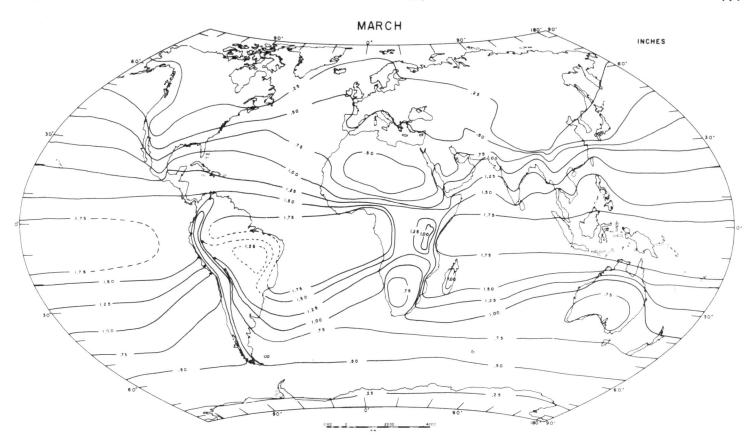


Figure 7.—Mean precipitable water—March.

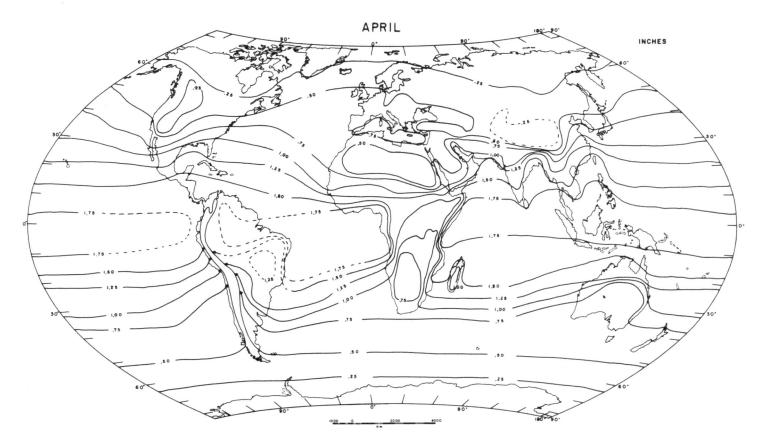


Figure 8.—Mean precipitable water—April.

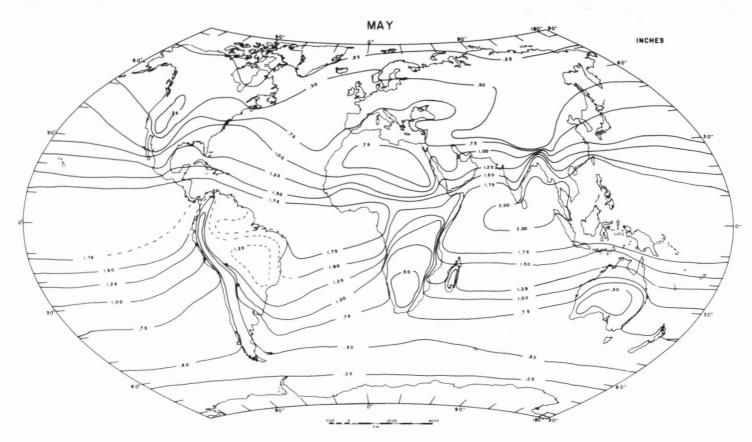


Figure 9.—Mean precipitable water—May.

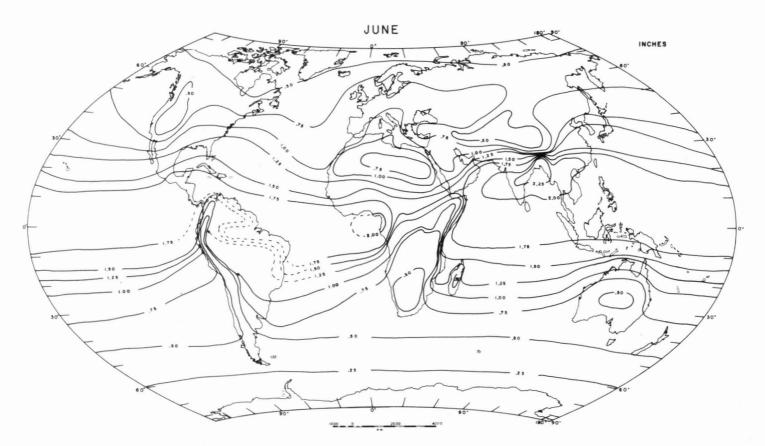


FIGURE 10.—Mean precipitable water—June.

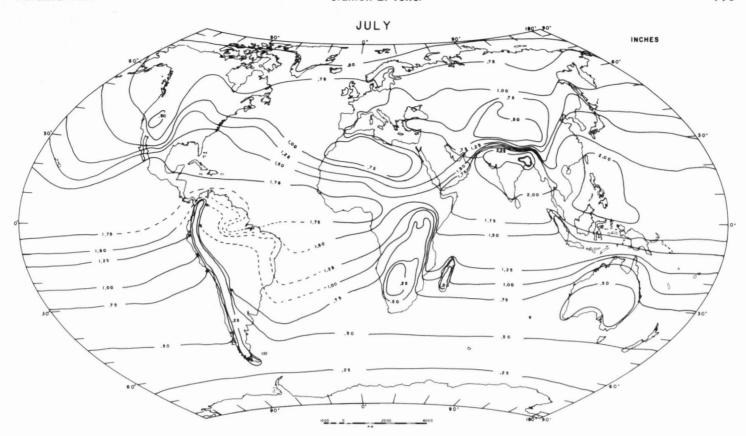


Figure 11.—Mean precipitable water—July.

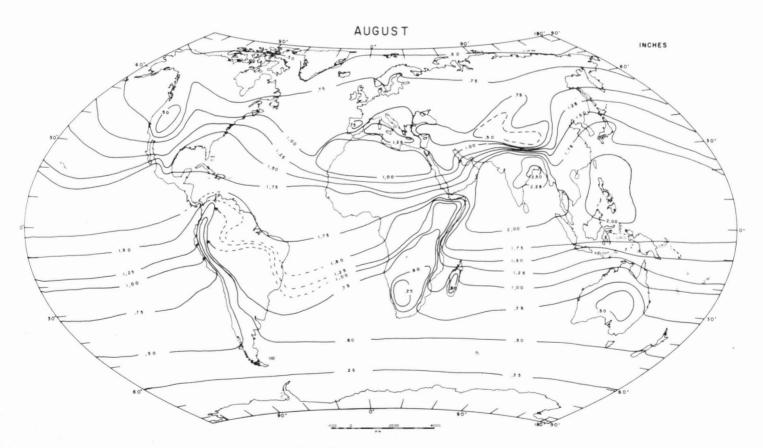


FIGURE 12.—Mean precipitable water—August.

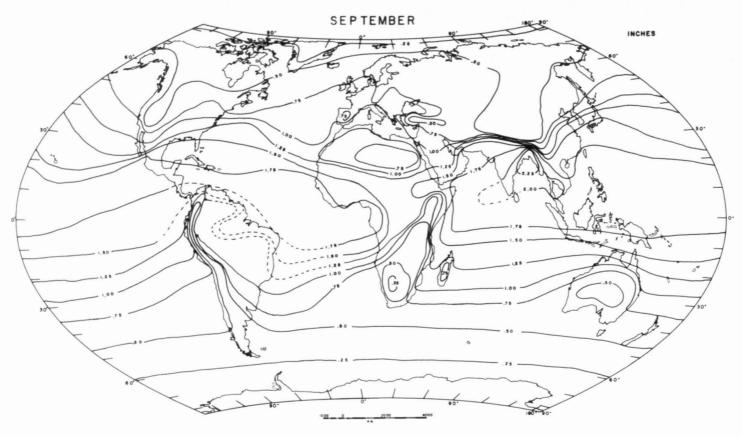


Figure 13.—Mean precipitable water—September.

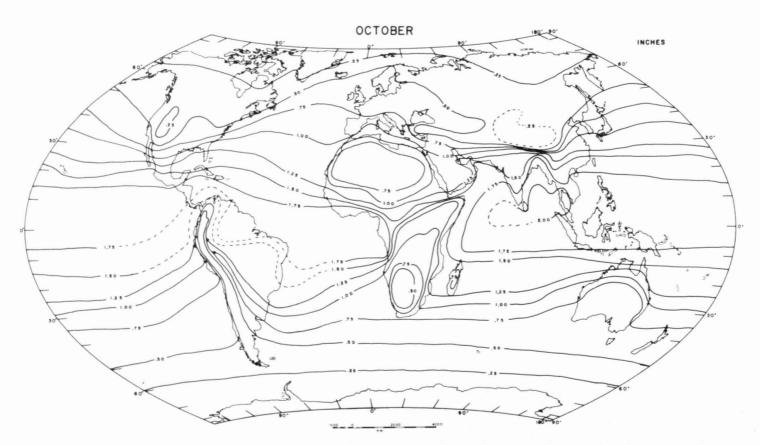


Figure 14.—Mean precipitable water—October.

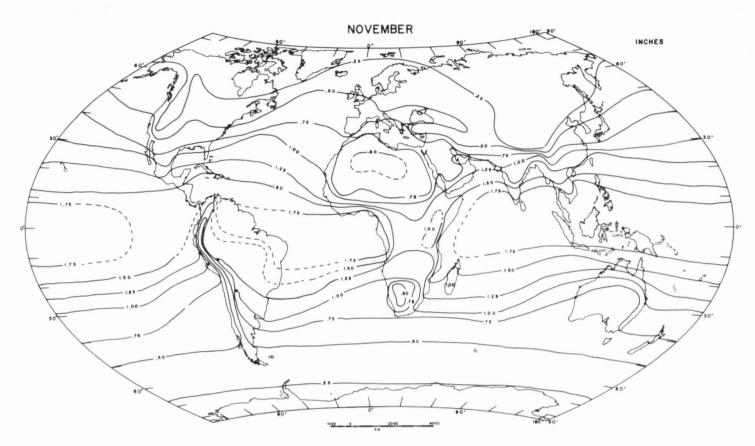


Figure 15.—Mean precipitable water—November.

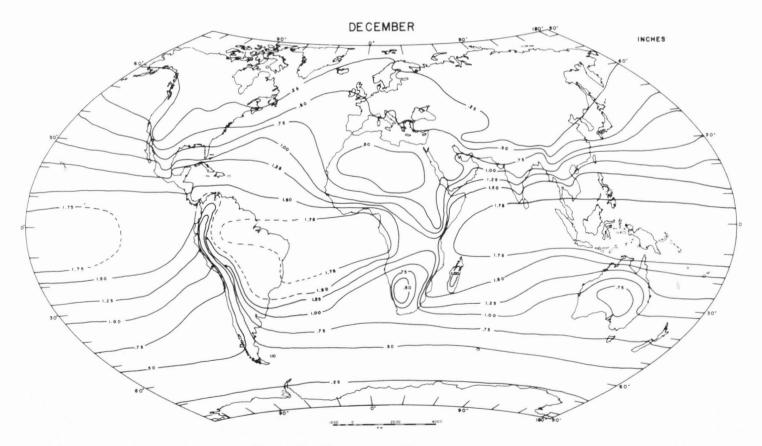


FIGURE 16.—Mean precipitable water—December.

strongly developed current cells, with their greater contrast between warm and cold portions, produce a more noticeable departure from a simple latitudinal pattern. The total annual amount of precipitable water is greater over the Southern Hemisphere. The Northern Hemisphere displays higher absolute values during the warm season and lower values during the cold season and, thus, a greater annual range.

The monthly maps (fig. 5–16) show the zones of precipitable water shift north and south following the sun and seasonal temperature changes. February and July (fig. 6 and 11) are the months of highest and lowest precipitable water at most stations. At some, August (fig. 12) replaces July, and at a smaller number, January (fig. 5) replaces February, however. In the spring and autumn, May and October (fig. 9 and 14) are the months that most closely approach the annual mean. Precipitable water, therefore, follows more closely the annual temperature cycle than the cycle of insolation.

During the course of the year, the lowest amounts of precipitable water are found at the Poles in the winter season. Also quite low are the continental interiors and mountain ranges during the winter. Here, actual amounts are hard to estimate because of the lack of data but are probably below 0.05 in.

The highest values for the year are found in the Bay of Bengal in July and August (fig. 11 and 12), during the summer monsoon. Here, values are over 2.50 in. The whole northern Indian Ocean and South China Sea show large means in this season. This appears as a large cell interrupted by the Indochina Peninsula.

An interesting seasonal pattern is revealed in the Bay of Bengal. In the winter months, values of precipitable water are moderate and center around 1.00 in. (see fig. 6). A latitudinal pattern is apparent. By April (fig. 8), a distinct bulge appears in the isolines and by May (fig. 9), a definite cell with over 2.00 in. of precipitable water has established itself. The cell strengthens in June (fig. 10) and begins to show a strong focus at the head of the Bay of Bengal as the summer monsoon arrives in this area. The peak development of the cell occurs in late July and early August (fig. 11 and 12) with values well over 2.50 in., and then begins a slow reversal. By November (fig. 15), only a slight bulge in the isolines remains.

An interesting way to compare the seasonal values is to look at the variety of annual ranges, both absolute and percentage, that are found. Narrow ranges are found over the equatorial oceans, high mountain ranges, and at higher latitudes. The equatorial oceans are areas of constantly high precipitable water. Both percentage and absolute ranges are low. The polar areas and high elevations show small absolute ranges, but the percentage variation can be quite wide due to the low mean amounts present in all seasons.

The greatest absolute range is found in the Bay of Bengal where the annual range is over 1.50 in. Largest percentage ranges are found in the interiors of the large continental masses of the Northern Hemisphere. In the Siberian region, winter values are around 0.10 in. and the summer values approach 1.00 in., an increase of almost tenfold. The range decreases toward the margins of the continents as the winter values of precipitable water become greater.

In general, the eastern sides of the continents have a greater range than the western sides. This is the result of both higher summer and lower winter totals. The contrast between windward and leeward positions with respect to the prevailing westerly winds in the middle latitudes does much to account for this difference.

4. CONCLUDING REMARKS

The foregoing discussion shows that the general pattern of the world distribution of precipitable water vapor is as might be expected. The distribution is not an isolated phenomenon but is closely related with many other surface and atmospheric variables. A closer analysis of the pattern with an emphasis on causal relations would readily reveal the effects of surface and air temperature, net radiation, elevation, distance from the sea, location with respect to prevailing winds, and areas of high and low pressure. The world patterns that are revealed generally tend to confirm the results of previous studies. In addition, the coverage has been extended in the present study to include new areas of the earth and new periods of time that were neglected in earlier works. The methods used in this study are simple and employ data that are readily available in almost any library. They have been shown to be reliable enough to be useful in other studies of this type or where the broad-scale average values of precipitable water are desired. It is hoped that in the future more refined studies on the distribution of precipitable water will be forthcoming. The use of expanded station coverage, computers, and detailed punchcard data will allow treatment of the problem at a much larger scale than was possible in this study. Perhaps these future efforts can concentrate on the areas that the current study has shown to be most in need of further research.

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CORRECTION NOTICE

Vol. 96, No. 1, January 1968, pp. 73-74: change these page numbers to read 72a and 72b respectively.